FNCE 926
Empirical Methods in CF

Lecture 1 – Linear Regression I

Professor Todd Gormley

Today's Agenda

- Introduction
- Discussion of Syllabus
- Review of linear regressions

About Me

- PhD from MIT economics
- Undergraduate at Michigan St. Univ.
- Research on bank entry & corporate topics involving risk and governance

Today's Agenda

- Introduction… about me
- Discussion of Syllabus
- Review of linear regressions

Course Objectives

- Provide toolbox & knowledge of cross-sectional & panel data empirical methods
- Course will have three-pronged approach
  - Lectures will provide you econometric ***intuition*** behind each method discussed
  - Course readings expose you to examples of these tools being used in recent research
  - Exercises will force you to use the methods taught in actual data

Reading Materials *[Part 1]*

- My lecture notes will be your primary source for each econometric tool
- But, please read background texts before lecture [see syllabus for relevant sections]
  - Angrist & Pischke's *Mostly Harmless*… book
  - Roberts & Whited (2010) paper
  - Greene's textbook on econometrics
  - Wooldridge's textbook on panel data

Reading Materials *[Part 2]*

- We will also be covering 35+ empirical papers; obtain these using Econlit or by going to authors' SSRN websites for working papers *[I've provided links]*
  - Sorry, for copyright reasons, I can't post the papers to Canvas…
  - Just let me know if you have any problem finding a particular paper

Study Groups

- 3 study groups will do in-class presentations
  - Choose own members; can change later if need to
  - Try to split yourself somewhat equally into groups
  - Choose initial groups <u>during today's break</u>; first group presentations will be in next class!
    [More about group presentations in a second…]

Course Structure

- Total of 150 possible points
  - In-class exam *[50 points]*
  - Five data exercises *[25 points]*
  - In-class presentations/participation *[25 points]*
  - Research proposal
    - Rough draft *[15 points]*
    - Final proposal *[35 points]*

Exam

- Done in last class, April 26
- More details when we get closer…, but a practice exam is already available on Canvas

Data exercises

- Exercises will ask you to download and manipulate data within Stata
  - E.g. will need to estimate a triple-diff
  - To receive credit, you will send me your DO files; I will then run them on my own dataset to confirm the coding is correct
  - More instructions in handouts [which will be available on Canvas website]

Turning in exercises

- Please upload both DO file and typed answers to Canvas; i.e., we won't be handing them in during class
  - They will be graded & returned on Canvas
  - Deadline to submit is noon
    [Canvas tracks when the file is uploaded]

In-class presentations & participation

- In every class (except today), students will present three papers in second half
  - Each study group does one presentation (this is why there needs to be three study groups)
    - But, only one student for each group actually presents
    - Rotate the presenter each week; doing this basically guarantees everyone full participation points
  - Assign papers for next class at end of class *[all papers are listed in the syllabus]*

PowerPoint Presentations *[Part 1]*

- Should last for 10 min., no more than 12 min.
  - Summarize *[2-3 minutes]*
  - Analytical discussion which should focus on identification and causality *[6-7 minutes]*
  - Conclusion *[1 minute]*
- Presentations followed by 5-10 minutes discussion; students must read all three papers
- <u>See handout on Canvas for more details</u>

PowerPoint Presentations *[Part 2]*

- Each student must also type up 2-3 sentence concern for each paper their group does NOT present and turn it in at start of class
  - I will randomly select one after each student presentation to further facilitate class discussion
  - Write your comment with one of these goals in mind…
    - Write down your own view of "biggest concern"
    - Or, write a concern you think presenter might miss!

Goal of Presentations

- Help you think critically about empirical tools discussed in previous class
- Allow you to see and learn from papers that actually use these techniques
- Gives you practice on presenting; this will be important in the long run

Research Proposal

- You will outline a possible empirical paper that uses tools taught in this course
  - Rough draft due March 22
  - Final proposal due exam week, May 3
- If you want, think of this as a jump start on a possible 2nd or 3rd year paper
- **See handout on Canvas for more details**

Office Hours & E-mail

- My office hours will be…
  - Thursdays, 1:30-3:00 p.m.
  - Or, by appointment
- Office location: 2458 SH-DH
- Email: tgormley@wharton.upenn.edu

Teaching Assistant

- The TA for this course will be…
  - Tetiana Davydiuk
  - davydiuk@wharton.upenn.edu
- She will be grading the exercises and answering any questions you might have about them

All other questions can be directed to me!

Tentative Schedule

- See syllabus…
- While exam date & final research proposal deadline are fixed, topics covered and other case due dates may change slightly if there is a sudden and unexpected class cancellation

How the course is structured…

- We will have a 1-2 lectures per 'tool'
  - I will lecture in first half (except today) on the 'tool'
  - In the second half of the following class, students will present papers using that particular tool

Canvas

- https://wharton.instructure.com
- Things available to download
  - Exercises & solutions *[after turned in]*
  - Lecture notes
  - Handouts that provide more details on what I expect for presentations & research proposal, including grading templates
  - Practice exam
  - Student presentations *[to help study for exam]*

Lecture Notes

- I will provide a copy of lecture notes on Canvas before the start of each class
  - I <u>strongly encourage</u> printing these out and bringing them with you to class!

Structural estimation lecture

- Prof. Taylor has agreed to give this lecture
- Tuesday, April 19… the usual time

Remaining Items

- 3 hours is long! We'll take one 10 minute break or two 5 minute breaks
- Read rest of syllabus for other details about the course including:
  - Class schedule or assigned papers are subject to change; I'll keep you posted of changes
  - Limitation of course; I won't have time to cover everything you should know, but it will be a good start

Questions

- If you have a question, ask! ☺
  - If you're confused, you're probably not alone
  - I don't mind being interrupted
  - If I'm going too fast, just let me know
- I may not always have an immediate answer, but all questions will be answered eventually
- **Any questions?**

Today's Agenda

- Introduction
- Discussion of Syllabus
- Review of linear regressions

My expectation is that you've seen most of this before; but it is helpful to review the key ideas that are useful in practice (without all the math)

Despite trying to do much of it without math; today's lecture likely to be long and tedious… (sorry)

Linear Regression – *Outline*

- The CEF and causality (very brief)
- Linear OLS model
- Multivariate estimation
- Hypothesis testing
- Miscellaneous issues

We will cover the latter two in the next lecture

Background readings

- Angrist and Pischke
  - *Sections 3.1-3.2, 3.4.1*
- Wooldridge
  - *Sections 4.1 & 4.2*
- Greene
  - *Chapter 3 and Sections 4.1-4.4, 5.7-5.9, 6.1-6.2*

Motivation

- Linear regression is arguably the most popular modeling approach in corporate finance
  - Transparent and intuitive
  - Very robust technique; easy to build on
  - Even if not interested in causality, it is useful for describing the data

Given importance, we will spend today & next lecture reviewing the key ideas

Motivation continued…

- As researchers, we are interested explaining how the world works
  - E.g. how are firms' choices regarding leverage are *explained* by their investment opportunities
    - I.e., if investment opportunities suddenly jumped for some random reason, how would we expect firms' leverage to respond on average?
  - More broadly, how is y explained by x, where both y and x are random variables?

Linear Regression – *Outline*

- The CEF and causality (very brief)
  - Random variables & the CEF
  - Using OLS to learn about the CEF
  - Briefly describe "causality"
- Linear OLS model
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A bit about random variables

- With this in mind, it is useful know that any random variable y can be written as

$$y = E(y \mid x) + \varepsilon$$

where (y, x, ε) are random variables and $E(\varepsilon \mid x)=0$

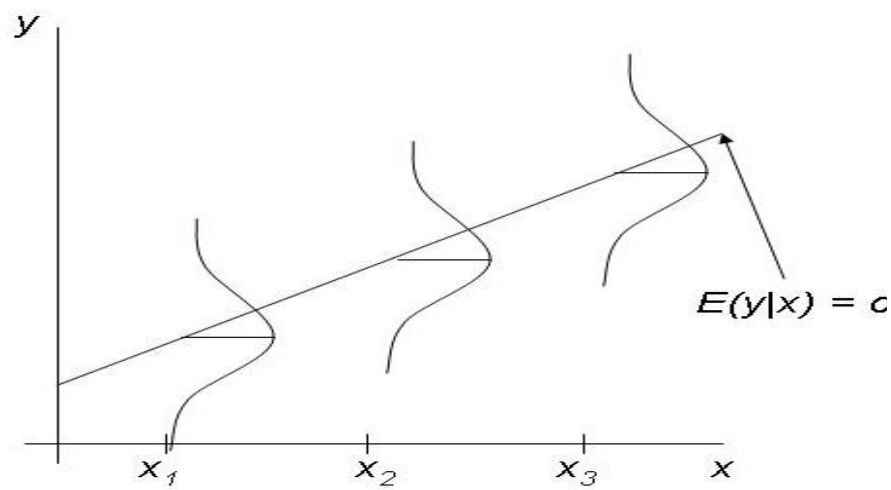
  - $E(y \mid x)$ is expected value of y given x
  - In words, y can be broken down into part 'explained' by x, $E(y \mid x)$, and a piece that is mean independent of x, ε

Conditional expectation function (CEF)

- $E(y|x)$ is what we call the CEF, and it has very desirable properties
  - Natural way to think about relationship between x and y
  - And, it is <u>best predictor of y given x</u> in a minimum mean-squared error sense
    - I.e. $E(y|x)$ minimizes $E[(y-m(x))^2]$,where $m(x)$ can be <u>any</u> function of x.

CEF visually…

- $E(y|x)$ is fixed, but unobservable



Our goal is to learn about the CEF

- Intuition: for any value of x, distribution of y is centered about $E(y|x)$

Linear Regression – *Outline*

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Linear regression and the CEF

- If done correctly, a linear regression can help us uncover what the CEF is
- Consider linear regression model, $y = \beta x + u$
  - y = dependent variable
  - x = independent variable
  - u = error term (or disturbance)
  - β = slope parameter

Some additional terminology

- Other terms for *y*…
  - Outcome variable
  - Response variable
  - Explained variable
  - Predicted variable
  - Regressand
- Other terms for *x*…
  - Covariate
  - Control variable
  - Explanatory variable
  - Predictor variable
  - Regressor

Details about $y = \beta x + u$

- (y, x, u) are random variables
- (y, x) are observable
- (u, β) are <u>un</u>observable
  - u captures everything that determines y after accounting for x *[This might be a lot of stuff!]*
  - We want to estimate β

Ordinary Least Squares (OLS)

- Simply put, OLS finds the β that minimizes the mean-squared error

$$\beta = \arg\min_{b} = E[(y - bx)^2]$$

- Using first order condition: E[x(y- β x)]=0, we have β =E(xy)/E(x^2)
- **Note:** <u>by definition</u>, the residual from this regression, y- β x, is uncorrelated with x

What great about this linear regression?

- It can be proved that…
  - βx is best* linear prediction of y given x
  - βx is best* linear approximation of $E(y|x)$

    * 'best' in terms of minimum mean-squared error

- This is quite useful. I.e. even if $E(y|x)$ is *nonlinear*, the regression gives us the best linear approximation of it

Linear Regression – *Outline*

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What about causality?

- Need to be careful here…
  - How x explains y, which this regression helps us understand, is not the same as learning the causal effect of x on y
  - For that, we need more assumptions…

The basic assumptions *[Part 1]*

- *Assumption #1:* $E(u) = 0$
  - With intercept, this is totally innocuous
  - Just change regression to $y = \alpha + \beta x + u$, where α is the intercept term
  - Now suppose, $E(u)=k\neq 0$
    - We could rewrite $u = k + w$, where $E(w)=0$
    - Then, model becomes $y = (\alpha + k) + \beta x + w$
    - Intercept is now just $\alpha + k$, and error, w, is mean zero
    - I.e. Any non-zero mean is absorbed by intercept

The basic assumptions *[Part 2]*

Intuition?

- *Assumption #2:* $E(u|x) = E(u)$
  - In words, average of u (i.e. unexplained portion of y) does not depend on value of x
  - This is "conditional mean independence" (CMI)
    - True if x and u are independent of each other
    - Implies u and x are uncorrelated

This is the key assumption being made when people make causal inferences

CMI Assumption

- Basically, assumption says you've got correct CEF model for <u>causal</u> effect of x on y
  - CEF is <u>causal</u> if it describes differences in average outcomes for a change in x
    - i.e. increase in x from values a to b is equal to $E(y|x=b)-E(y|x=a)$ **[In words?]**
  - Easy to see that this is only true if $E(u|x) = E(u)$ *[This is done on next slide…]*

Example of why CMI is needed

- With model $y = \alpha + \beta x + u$,
  - $E(y|x=a) = \alpha + \beta a + E(u|x=a)$
  - $E(y|x=b) = \alpha + \beta b + E(u|x=b)$
  - Thus, $E(y|x=b) - E(y|x=a) = \beta(b-a) + E(u|x=b) - E(u|x=a)$
  - This only equals what we think of as the 'causal' effect of x changing from a to b if $E(u|x=b) = E(u|x=a)$… i.e. CMI assumption holds

Tangent – CMI versus correlation

- CMI (which implies *x* and *u* are uncorrelated) is needed for <u>no bias</u> *[which is a finite sample property]*
- But, we only need to assume a zero correlation between *x* and *u* for <u>consistency</u> *[which is a large sample property]*
  - More about bias *vs.* consistency later; but we typically care about consistency, which is why I'll often refer to correlations rather than CMI

Is it plausible?

- Admittedly, there are many reasons why this assumption might be violated
  - Recall, u captures all the factors that affect y other than x… It will contain a lot!
  - Let’s just do a couple of examples…

Ex. #1 – Capital structure regression

- Consider following firm-level regression:

$$Leverage_i = \alpha + \beta Profitability_i + u_i$$

  - CMI implies average *u* is same for each profitability
  - Easy to find a few stories why this isn't true…
    - #1 – unprofitable firms tend to have higher bankruptcy risk, which by tradeoff theory, should mean a lower leverage
    - #2 – unprofitable firms have accumulated less cash, which by pecking order means they should have more leverage

Ex. #2 – Investment

Measure of investment opportunities

- Consider following firm-level regression:

$$Investment_i = \alpha + \beta Q_i + u_i$$

  - CMI implies average u is same for each Tobin's Q
  - Easy to find a few stories why this isn't true…
    - #1 – Firms with low Q might be in distress & invest less
    - #2 – Firms with high Q might be smaller, younger firms that have a harder time raising capital to fund investments

Is there a way to test for CMI?

- Let $\hat{y}$ be the predicted value of *y*, i.e. $\hat{y} = \alpha + \beta x$, where α and β are OLS estimates
- And, let $\hat{u}$ be the residual, i.e. $\hat{u} = y - \hat{y}$
- Can we prove CMI if residuals if $E(\hat{u})=0$ and if $\hat{u}$ is uncorrelated with x?
  - **Answer:** No! By construction these residuals are mean zero and uncorrelated with *x*. See earlier derivation of OLS estimates

Identification police

- What people call the "identification police" are those that look for violations of CMI
  - I.e. the "police" look for a reason why the model's disturbance is correlated with x
    - Unfortunately, it's not that hard…
    - **Trying to find ways to ensure the CMI assumption holds and causal inferences can be made will be a key focus of this course**

A side note about "endogeneity"

- Many "police" will criticize a model by saying it has an "endogeneity problem" but then don't say anything further…

- **But what does it mean to say there is an "an endogeneity problem"?**

A side note about "endogeneity"

- **My view:** such vague "endogeneity" critics suspect something is potentially wrong, but don't really know why or how
  - **Don't let this be you! Be specific about what the problem is!**

- Violations to CMI can be roughly categorized into three bins… which are?

Three reasons why CMI is violated

- Omitted variable bias
- Measurement error bias
- Simultaneity bias
  - We will look at each of these in much more detail in the "Causality" lecture

What "endogenous" means to me

- An "endogenous" x is when its value depends on y (i.e. it determined jointly with y such that there is simultaneity bias).
  - But, some use a broader definition to mean any correlation between x and u *[e.g. Roberts & Whited (2011)]*
  - Because of the confusion, I avoid using "endogeneity"; I'd recommend the same for you
    - I.e. Be specific about CMI violation; just say omitted variable, measurement error, or simultaneity bias

A note about presentations…

- Think about "causality" when presenting next week and the following week
  - I haven't yet formalized the various reasons for why "causal" inferences shouldn't be made; but I'd like you to take a stab at thinking about it

Linear Regression – *Outline*

- The CEF and causality (very brief)
- Linear OLS model
  - Basic interpretation
  - Rescaling & shifting of variables
  - Incorporating non-linearities
- Multivariate estimation
- Hypothesis testing
- Miscellaneous issues

Interpreting the estimates

- Suppose I estimate the following model of CEO compensation

$$salary_i = \alpha + \beta ROE_i + u_i$$

  - Salary for CEO *i* is in \$000s; ROE is a %

- If you get… $\hat{\alpha} = 963.2$

$$\hat{\beta} = 18.50$$

  - What do these coefficients tell us?
  - Is CMI likely satisfied?

Interpreting the estimates – *Answers*

$$salary_i = 963.2 + 18.5 ROE_i + u_i$$

- What do these coefficients tell us?
  - 1 percentage point increase in ROE is associated with $18,500 increase in salary
  - Average salary for CEO with ROE = 0 was equal to $963,200
- Is CMI likely satisfied? **Probably not**

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Scaling the *dependent variable*

- What if I change measurement of salary from \$000s to \$s by multiplying it by 1,000?
  - Estimates were… $\hat{\alpha} = 963.2$

    $\hat{\beta} = 18.50$
  - Now, they will be… $\hat{\alpha} = 963,200$

    $\hat{\beta} = 18,500$

Scaling y continued…

- Scaling y by an amount c just causes all the estimates to be scaled by the same amount
  - Mathematically, easy to see why…

$$y = \alpha + \beta x + u$$

$$cy = (c\alpha) + (c\beta)x + cu$$

New intercept

New slope

Scaling *y* continued…

- Notice, the scaling has *no* effect on the relationship between ROE and salary
  - I.e. because *y* is expressed in \$s now, $\hat{\beta} = 18{,}500$ means that a one percentage point increase in ROE is still associated with \$18,500 increase in salary

Scaling the *independent variable*

- What if I instead change measurement of ROE from percentage to decimal? (i.e. multiply ROE by 1/100)
  - Estimates were…
    $$\hat{\alpha} = 963.2$$
    $$\hat{\beta} = 18.50$$
  - Now, they will be…
    $$\hat{\alpha} = 963.2$$
    $$\hat{\beta} = 1{,}850$$

Scaling x continued…

- Scaling x by an amount k just causes the slope on x to be scaled by $1/k$
  - Mathematically, easy to see why…

$$y = \alpha + \beta x + u$$

$$y = \alpha + \underbrace{\left(\frac{\beta}{k}\right)}_{\text{New slope}} kx + u$$

Will interpretation of estimates change?

Answer: Again, no!

Scaling both x and y

- If scale y by an amount c and x by amount k, then we get…
  - Intercept scaled by c
  - Slope scaled by c/k

$$y = \alpha + \beta x + u$$

$$cy = (c\alpha) + \left(\frac{c\beta}{k}\right)kx + cu$$

- **When is scaling useful?**

Practical application of scaling **#1**

- No one wants to see a coefficient of 0.000000456 **or** 1,234,567,890
- Just scale the variables for cosmetic purposes!
  - It will effect coefficients & SEs
  - But, it won't affect t-stats or inference

Practical application of scaling **#2** *[P1]*

- To improve interpretation, in terms of found magnitudes, helpful to scale by the variables by their sample standard deviation
  - Let σ_x and σ_y be sample standard deviations of x and y respectively
  - Let c, the scalar for y, be equal to $1/\sigma_y$
  - Let k, the scalar for x, be equal to $1/\sigma_x$
  - I.e. unit of x and y is now standard deviations

Practical application of scaling **#2** *[P2]*

- With the prior rescaling, how would we interpret a slope coefficient of 0.25?
  - **Answer** = a 1 s.d. increase in x is associated with ¼ s.d. increase in y
  - The slope tells us how many standard deviations y changes, on average, for a standard deviation change in x
  - Is 0.25 large in magnitude? What about 0.01?

Shifting the variables

- Suppose we instead add c to y and k to x (i.e. we shift y and x up by c and k respectively)

- Will the estimated slope change?

Shifting continued…

- No! Only the estimated intercept will change
  - Mathematically, easy to see why…

$$y = \alpha + \beta x + u$$
$$y + c = \alpha + c + \beta x + u$$
$$y + c = \alpha + c + \beta(x + k) - \beta k + u$$
$$y + c = (\alpha + c - \beta k) + \beta(x + k) + u$$

New intercept

Slope the same

Practical application of shifting

- To improve interpretation, sometimes helpful to demean x by its sample mean
  - Let μ_x be the sample mean of x; regress y on x - μ_x
  - Intercept now reflects expected value of y for $x = \mu_x$

$$y = \left(\alpha + \beta\mu_x\right) + \beta\left(x - \mu_x\right) + u$$

$$E(y \mid x = \mu_x) = \left(\alpha + \beta\mu_x\right)$$

  - This will be very useful when we get to diff-in-diffs

Break Time

- Let’s take a 10 minute break

Linear Regression – *Outline*

- The CEF and causality (very brief)
- Linear OLS model
  - Basic interpretation
  - Rescaling & shifting of variables
  - Incorporating non-linearities
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Incorporating nonlinearities *[Part 1]*

- Assuming that the causal CEF is linear may not always be that realistic
  - E.g. consider the following regression

$$wage = \alpha + \beta \, education + \mathbf{u}$$

  - Why might a linear relationship between # of years of education and level of wages be unrealistic? How can we fix it?

Incorporating nonlinearities *[Part 2]*

- Better assumption is that each year of education leads to a constant proportionate (i.e. percentage) increase in wages
  - Approximation of this intuition captured by…

$$ln(wage) = \alpha + \beta\, education + \mathbf{u}$$

  - I.e. the linear specification is very flexible because it can capture linear relationships between non-linear variables

Common nonlinear function forms

- Regressing Levels on Logs
- Regressing Logs on Levels
- Regressing Logs on Logs

Let's discuss how to interpret each of these

The usefulness of log

- Log variables are useful because $100 * \Delta \ln(y) \approx \% \ \Delta y$
  - **Note:** When I (and others) say "Log", we really mean the natural logarithm, "Ln". E.g. if you use the "log" function in Stata, it assumes you meant "ln"

Interpreting log-level regressions

- If estimate, the ln(wage) equation, 100 β will tell you the % Δ wage for an additional year of education. To see this…

$$\ln(wage) = \alpha + \beta education + u$$

$$\Delta \ln(wage) = \beta \Delta education$$

$$100 \times \Delta \ln(wage) = (100\beta)\Delta education$$

$$\%\Delta wage \approx (100\beta)\Delta education$$

Log-level interpretation continued…

- The proportionate change in y for a given change in x is assumed constant
  - The change in y is not assumed to be constant… it gets larger as x increases
  - Specifically, $\ln(y)$ is assumed to be linear in x; but y is <u>not</u> a linear function of x…

$$\ln(y) = \alpha + \beta x + u$$

$$y = \exp(\alpha + \beta x + u)$$

Example interpretation

- Suppose you estimated the wage equation (where wages are \$/hour) and got…

$$\mathit{ln(wage)} = \mathbf{0.584 + 0.083}\,\mathit{education}$$

  - What does an additional year of education get you?

Answer = 8.3% increase in wages.

  - Any potential problems with the specification?
  - Should we interpret the intercept?

Interpreting *log-log* regressions

- If estimate the following…

$$\ln(y) = \alpha + \beta \ln(x) + u$$

- β is the **elasticity** of y w.r.t. x!
  - i.e. β is the percentage change in y for a percentage change in x
  - **Note:** regression assumes constant elasticity between y and x regardless of level of x

Example interpretation of log-log

- Suppose you estimated the CEO salary model using logs got the following:

$$ln(salary) = \mathbf{4.822 + 0.257}\, ln(sales)$$

- What is the interpretation of 0.257?

Answer = For each 1% increase in sales, salary increases by 0.257%

Interpreting *level-log* regressions

- If estimate the following…

$$y = \alpha + \beta \ln(x) + u$$

- $\beta/100$ is the change in y for 1% change x

Example interpretation of level-log

- Suppose you estimated the CEO salary model using logs got the following, where salary is expressed in $000s:

$$\textit{salary} = \mathbf{4.822 + 1{,}812.5}\,\textit{ln(sales)}$$

- What is the interpretation of 1,812.5?

  Answer = For each 1% increase in sales, salary increases by $18,125

Summary of log functional forms

Model	Dependent Variable	Independent Variable	Interpretation of β
Level-Level	y	x	$dy = \beta dx$
Level-Log	y	ln(x)	$dy = (\beta/100)\%dx$
Log-Level	ln(y)	x	$\%dy = (100\beta)dx$
Log-Log	ln(y)	ln(x)	$\%dy = \beta\%dx$

- Now, let's talking about what happens if you change units (i.e. scale) for either y or x in these regressions…

Rescaling logs doesn't matter *[Part 1]*

- What happens to intercept & slope if rescale (i.e. change units) of *y* when in log form?
- **Answer** = Only intercept changes; slope unaffected because it measures proportional change in *y* in Log-Level model

$$\log(y) = \alpha + \beta x + u$$

$$\log(c) + \log(y) = \log(c) + \alpha + \beta x + u$$

$$\log(cy) = \left(\log(c) + \alpha\right) + \beta x + u$$

Rescaling logs doesn't matter *[Part 2]*

- Same logic applies to changing scale of x in level-log models… only intercept changes

$$y = \alpha + \beta \log(x) + u$$

$$y + \beta \log(c) = \alpha + \beta \log(x) + \beta \log(c) + u$$

$$y = \left(\alpha - \beta \log(c)\right) + \beta \log(cx) + u$$

Rescaling logs doesn't matter *[Part 3]*

- **Basic message** – If you rescale a logged variable, it will not effect the slope coefficient because you are only looking at proportionate changes

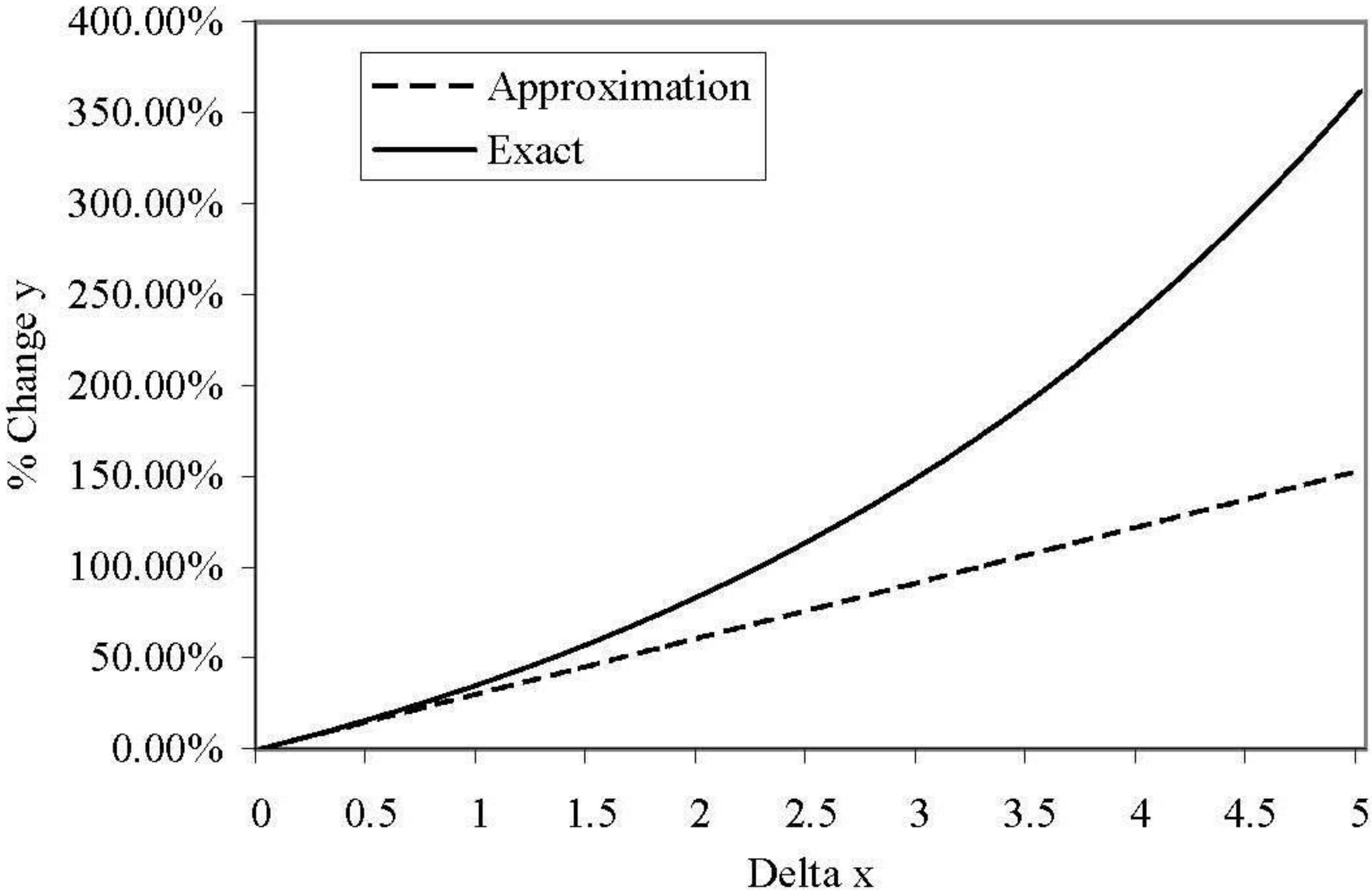
Log approximation problems

- I once discussed a paper where author argued that allowing capital inflows into country caused -120% change in stock prices during crisis periods…
  - **Do you see a problem with this?**
    - Of course! A 120% drop in stock prices isn't possible. The true percentage change was -70%. Here is where that author went wrong…

Log approximation problems *[Part 1]*

- Approximation error occurs because as true $\%\Delta y$ becomes larger, $100\Delta\ln(y) \approx \%\Delta y$ becomes a worse approximation
- To see this, consider a change from y to y' …
  - **Ex. #1:** $\frac{y'-y}{y} = 5\%$, and $100\Delta\ln(y) = 4.9\%$
  - **Ex. #2:** $\frac{y'-y}{y} = 75\%$, <u>but</u> $100\Delta\ln(y) = 56\%$

Log approximation problems *[Part 2]*



Log approximation problems *[Part 3]*

- Problem also occurs for negative changes
  - **Ex. #1:** $\dfrac{y'-y}{y} = -5\%$, and 100Δln(y) = -5.1%
  - **Ex. #2:** $\dfrac{y'-y}{y} = -75\%$, <u>but</u> 100Δln(y)= -139%

Log approximation problems *[Part 4]*

- So, if implied percent change is large, better to convert it to true % change before interpreting the estimate

$$\ln(y) = \alpha + \beta x + u$$

$$\ln(y') - \ln(y) = \beta(x' - x)$$

$$\ln(y'/y) = \beta(x' - x)$$

$$y'/y = \exp\left(\beta(x' - x)\right)$$

$$[(y' - y)/y]\% = 100\left[\exp\left(\beta(x' - x)\right) - 1\right]$$

Log approximation problems *[Part 5]*

- We can now use this formula to see what true % change in y is for $x'-x = 1$

$$[(y'-y)/y]\% = 100\left[\exp\left(\beta(x'-x)\right)-1\right]$$

$$[(y'-y)/y]\% = 100\left[\exp\left(\beta\right)-1\right]$$

  - If $\beta = 0.56$, the percent change isn't 56%, it is

$$100\left[\exp\left(0.56\right)-1\right] = 75\%$$

Recap of last two points on logs

- Two things to keep in mind about using logs
  - Rescaling a logged variable doesn't affect slope coefficients; it will only affect intercept
  - Log is only approximation for % change; it can be a very bad approximation for large changes

Usefulness of logs – **Summary**

- Using logs gives coefficients with appealing interpretation
- Can be ignorant about unit of measurement of log variables since they're proportionate Δ s
- Logs of y or x can mitigate influence of outliers

"Rules of thumb" on when to use logs

- Helpful to take logs for variables with…
  - Positive currency amount
  - Large integral values (e.g. population)
- Don't take logs for variables measured in years or as proportions
- If $y \in [0,\infty)$, can take $\ln(1+y)$, but be careful… nice interpretation <u>no</u> longer true…

What about using ln(1+y)?

- Because ln(0) doesn't exist, people use ln(1+*y*) for non-negative variables, i.e. $y \in [0,\infty)$
  - Be careful interpreting the estimates! Nice interpretation no longer true, especially if a lot of zeros or many small values in *y* *[Why?]*
    - *Ex. #1: What does it mean to go from ln(0) to ln(x>0)?*
    - *Ex. #2: And, Ln(x' +1) – Ln(x+1) is not percent change of x*
  - In this case, might be better to scale *y* by another variable instead, like firm size

Tangent – Percentage Change

- What is the percent change in unemployment if it goes from 10% to 9%?
  - This is 10 percent drop
  - It is a 1 percentage point drop
    - **Percentage change** is $[(x_1 - x_0)/x_0]\times 100$
    - **Percentage point change** is the raw change in percentages

Please take care to get this right in description of your empirical results

Models with quadratic terms *[Part 1]*

- Consider $y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$
- Partial effect of x is given by…

$$\Delta y = \left(\beta_1 + 2\beta_2 x\right)\Delta x$$

  - What is different about this partial effect relative to everything we've seen thus far?
    - **Answer** = It depends on the value of x. So, we will need to pick a value of x to evaluation (e.g. $\bar{x}$)

Models with quadratic terms *[Part 2]*

- If $\hat{\beta}_1 > 0, \hat{\beta}_2 < 0$, then it has parabolic relation
  - Turning point = Maximum = $\left|\hat{\beta}_1 / 2\hat{\beta}_2\right|$
  - *Know where this turning point is!* Don't claim a parabolic relation if it lies outside range of *x*!
  - Odd values might imply misspecification or simply mean the quadratic terms are irrelevant and should be excluded from the regression

Linear Regression – *Outline*

- The CEF and causality (very brief)
- Linear OLS model
- Multivariate estimation
  - Properties & Interpretation
  - Partial regression interpretation
  - R^2, bias, and consistency
- Hypothesis testing
- Miscellaneous issues

Motivation

- Rather uncommon that we have just one independent variable
    - So, now we will look at multivariate OLS models and their properties…

Basic multivariable model

- Example with constant and k regressors

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + u$$

- Similar identifying assumptions as before
  - No collinearity among covariates **[why?]**
  - $E(u | x_1, \ldots, x_k) = 0$
    - Implies no correlation between any x and u, which means we have the correct model of the true causal relationship between y and $(x_1, \ldots, x_k)$

Interpretation of estimates

- Estimated intercept, $\hat{\beta}_0$, is predicted value of *y* when all *x* = 0; sometimes this makes sense, sometimes it doesn't
- Estimated slopes, $\left(\hat{\beta}_1, \ldots, \hat{\beta}_k\right)$, have a more subtle interpretation now…

$$y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots + \hat{\beta}_k x_k + \hat{u}$$

  - How would you interpret $\hat{\beta}_1$?

Interpretation – *Answer*

- Estimated slopes, $\left(\hat{\beta}_1, \ldots, \hat{\beta}_k\right)$, have partial effect interpretations
- Typically, we think about change in just one variable, e.g. Δx_1, holding constant all other variables, i.e. ($\Delta x_2, \ldots, \Delta x_k$ all equal 0)
  - This is given by $\Delta\hat{y} = \hat{\beta}_1 \Delta x_1$
  - I.e. $\hat{\beta}_1$ is the coefficient holding *all else fixed* (ceteris paribus)

Interpretation continued…

- But, can also look at how changes in multiple variables at once affects predicted value of y
  - I.e. given changes in x_1 through x_k we obtain the predicted change in y, Δy

$$\Delta \hat{y} = \hat{\beta}_1 \Delta x_1 + \ldots + \hat{\beta}_k \Delta x_k$$

Example interpretation – College GPA

- Suppose we regress college GPA onto high school GPA (4-point scale) and ACT score for N = 141 university students

$$colGPA = 1.29 + 0.453 hsGPA + 0.0094 ACT$$

  - **What does the intercept tell us?**
  - **What does the slope on *hsGPA* tell us?**

Example – Answers

- Intercept pretty meaningless… person with zero high school GPA and ACT doesn't exist
- Example interpretation of slope…
  - Consider two students, Ann and Bob, with identical ACT score, but Ann's GPA is 1 point higher than Bob. Best prediction of Ann's college GPA is that it will be 0.453 higher than Bob's

Example continued…

- Now, what is effect of increasing high school GPA by 1 point and ACT by 1 point?

$$\Delta colGPA = 0.453 \times \Delta hsGPA + 0.0094 \times \Delta ACT$$

$$\Delta colGPA = 0.453 + 0.0094$$

$$\Delta colGPA = 0.4624$$

Example continued…

- Lastly, what is effect of increasing high school GPA by 2 points and ACT by 10 points?

$$\Delta colGPA = 0.453 \times \Delta hsGPA + 0.0094 \times \Delta ACT$$

$$\Delta colGPA = 0.453 \times 2 + 0.0094 \times 10$$

$$\Delta colGPA = 1$$

Fitted values and residuals

- Definition of residual for observation i, $\hat{u}_i$

$$\hat{u}_i = y_i - \hat{y}_i$$

- Properties of residual and fitted values
  - Sample average of residuals = 0; implies that sample average of $\hat{y}$ equals sample average of y
  - Sample covariance between each independent variable and residuals = 0
  - Point of means $(\bar{y}, \bar{x}_1, \ldots, \bar{x}_k)$ lies on regression line

Tangent about residuals

- Again, it bears repeating…
  - Looking at whether the residuals are correlated with the x's is NOT a test for causality
  - By construction, they are uncorrelated with x
  - There is no "test" of whether the CEF is the causal CEF; that justification will need to rely on <u>economic</u> arguments

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Question to motivate the topic…

- **What is wrong with the following? And why?**
  - Researcher wants to know effect of x on y after controlling for z
  - So, researcher removes the variation in y that is driven by z by regressing y on z & saves residuals
  - Then, researcher regresses these residuals on x and claims to have identified effect of x on y controlling for z using this regression

We'll answer why it's wrong in a second…

Partial regression *[Part 1]*

- The following is quite useful to know…
- Suppose you want to estimate the following

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

  - Is there another way to get $\hat{\beta}_1$ that doesn't involve estimating this directly?
    - **Answer:** Yes! You can estimate it by regressing the residuals from a regression of y on x_2 onto the residuals from a regression of x_1 onto x_2

Partial regression *[Part 2]*

- To be clear, you get $\hat{\beta}_1$, by…

  #1 – Regress y on x_2; save residuals (call them $\tilde{y}$)

  #2 – Regress x_1 on x_2; save residuals (call them $\tilde{x}$)

  #3 – Regress $\tilde{y}$ onto $\tilde{x}$; the estimated coefficient will be the same as if you'd just run the original multivariate regression!!!

Partial regression – *Interpretation*

- Multivariate estimation is basically finding effect of each independent variable after partialing out effect of other variables
  - I.e. Effect of x_1 on y after controlling for x_2, (i.e. what you'd get from regressing y on both x_1 and x_2) is the same as what you get after you partial out the effect x_2 from both x_1 and y and then run a regression using the residuals

Partial regression – *Generalized*

- This property holds more generally…
  - Suppose X_1 is vector of independent variables
  - X_2 is vector of more independent variables
  - And, you want to know that coefficients on X_1 that you would get from a multivariate regression of *y* onto all the variables in X_1 and X_2…

Partial regression – *Generalized, Part 2*

- You can get the coefficients for each variable in X_1 by…
  - Regress *y* and each variable in X_1 onto all the variables in X_2 (at once), save residuals from each regression
  - Do a regression of residuals; i.e. regress *y* onto variables of X_1, but replace *y* and X_1 with the residuals from the corresponding regression in step #1

Practical application of partial regression

- **Now, what is wrong with the following?**
  - Researcher wants to know effect of x on y after controlling for z
  - So, researcher removes the variation in y that is driven by z by regressing y on z & saves residuals
  - Then, researcher regresses these residuals on x and claims to have identified effect of x on y controlling for z using this regression

Practical application – Answer

- It's wrong because it didn't partial effect of z out of x! Therefore, it is NOT the same as regressing y onto both x and z!
- Unfortunately, it is commonly done by researchers in finance *[e.g. industry-adjusting]*
  - We will see how badly this can mess up things in a later lecture where we look at my paper with David Matsa on unobserved heterogeneity

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Goodness-of-Fit (R^2)

- A lot is made of R-squared; so let's quickly review exactly what it is
- Start by defining the following:
  - Sum of squares total (SST)
  - Sum of squares explained (SSE)
  - Sum of squares residual (SSR)

Definition of SST, SSE, SST

If N is the number of observations and the regression has a constant, then

$$SST = \sum_{i=1}^{N} \left(y_i - \overline{y} \right)^2$$

SST is total variation in *y*

$$SSE = \sum_{i=1}^{N} \left(\hat{y}_i - \overline{y} \right)^2$$

SSE is total variation in predicted *y*
[mean of predicted y = mean of y]

$$SSR = \sum_{i=1}^{N} \hat{u}_i^2$$

SSR is total variation in residuals
[mean of residual = 0]

SSR, SST, and SSE continued…

- The total variation, SST, can be broken into two pieces… the explained part, SSE and unexplained part, SSR

$$\mathbf{SST = SSE + SSR}$$

- R^2 is just the share of total variation that is explained! In other words,

$$\mathbf{R^2 = SSE/SST = 1 - SSR/SST}$$

More about R^2

- As seen on last slide, R^2 must be between 0 and 1
- It can also be shown that R^2 is equal to the square of the correlation between *y* and predicted *y*
- If you add an independent variable, R^2 will never go down

Adjusted R^2

- Because R^2 always goes up, we often use what is called Adjusted R^2

$$AdjR^2 = 1 - \left(1 - R^2\right)\left(\frac{N-1}{N-1-k}\right)$$

  - k = # of regressors, excluding the constant
  - Basically, you get penalized for each additional regressor, such that adjusted R^2 won't go up after you add another variable if it doesn't improve fit much [it can actually go down!]

Interpreting R^2

- If I tell you the R^2 is 0.014 from a regression, what does that mean? Is it bad?
  - **Answer #1** = It means I'm only explaining about 1.4% of the variation in *y* with the regressors that I'm including in the regression
  - **Answer #2** = Not necessarily! It doesn't mean the model is wrong; you might still be getting a consistent estimate of the β you care about!

Unbiasedness *versus* Consistency

- When we say an estimate is unbiased or consistent, it means we think it has a causal interpretation…
  - I.e. the CMI assumption holds and the x's are all uncorrelated with the disturbance, u
- **Bias** refers to finite sample property; **consistency** refers to asymptotic property

More formally…

- An estimate, $\hat{\beta}$, is <u>unbiased</u> if $E\left(\hat{\beta}\right) = \beta$
  - I.e. on average, the estimate is centered around the true, unobserved value of β
  - Doesn't say whether you get a more precise estimate as sample size increases
- An estimate is <u>consistent</u> if $\underset{N \to \infty}{plim}\, \hat{\beta} = \beta$
  - I.e. as sample size increases, the estimate converges (in probability limit) to the true coefficient

Unbiasedness of OLS

- OLS will be unbiased when…
  - Model is linear in parameters
  - We have a random sample of x
  - No perfect collinearity between x's
  - $\mathrm{E}(u \mid x_1, \ldots, x_k) = 0$
    [Earlier assumptions #1 and #2 give us this]
- Unbiasedness is nice feature of OLS; but in practice, we care more about consistency

Consistency of OLS

- OLS will be consistent when
  - Model is linear in parameters
  - u is not correlated with any of the x's,

    [CMI assumptions #1 and #2 give us this]
- Again, this is good
- See textbooks for more information

Summary of Today *[Part 1]*

- The CEF, $E(y|x)$ has desirable properties
  - Linear OLS gives best linear approx. of it
  - If correlation between error, u, and independent variables, x's, is zero it has causal interpretation
- Scaling & shifting of variables doesn't affect inference, but can be useful
  - E.g. demean to give intercepts more meaningful interpretation or rescale for cosmetic purposes

Summary of Today *[Part 2]*

- Multivariate estimates are partial effects
  - I.e. effect of x_1 holding $x_2,\ldots, x_k$ constant
  - Can get same estimates in two steps by first partialing out some variables and regressing residuals on residuals in second step

Assign papers for next week…

- Angrist (AER 1990)
  - Military service & future earnings
- Angrist and Lavy (QJE 1999)
  - Class size & student achievements
- Acemoglu, et al. (AER 2001)
  - Institutions and economic development

These seminal papers in economics with clever identification strategies… i.e., what we aspire to learn about later in the course

In First Half of Next Class

- Finish discussion of the linear regression
  - Hypothesis testing
  - Irrelevant regressors & multicollinearity
  - Binary variables & interactions
- Relevant readings; see syllabus